Normal Deduction in the Intuitionistic Linear Logic

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Abstract

A natural deduction system **NDIL** described here admits normalization and has subformula property. It has standard axioms $A \vdash A$, $\vdash 1$, standard introduction and elimination rules for $\&, \multimap$ (linear implication), $\oplus$ and quantifiers. The rules for $\otimes$ are now standard too. Structural rules are (implicit) permutation plus contraction and weakening for m-formulas. The rules for ! use an idea of D. Prawitz. By a m-formula we mean 1, any formula beginning with !, and any expression $<\Gamma>_{!A}$, where Γ is a list of formulas and m-formulas, and A is a formula. Derivable objects are sequents $\Gamma \vdash A$ where Γ is a multiset of formulas and m-formulas, and A is a formula. The rules for !, weakening and contraction are as follows:

$$<> I\frac{\Gamma \vdash !A}{<\Gamma>_{!A} \vdash !A} \quad !E\frac{\Gamma \vdash !A}{\Gamma \vdash A} \quad !I\frac{\mathbf{\Gamma} \vdash A}{\mathbf{\Gamma} \vdash !A} \quad <> E\frac{<\Gamma>_{!B}, \Sigma \vdash A}{\Gamma, \Sigma \vdash A} \quad weak\frac{\Gamma \vdash \mathbf{A} \quad \Sigma \vdash B}{\Gamma, \Sigma \vdash B} \quad c\frac{\mathbf{B}, \mathbf{B}, \Gamma \vdash D}{\mathbf{B}, \Gamma \vdash D}$$

provided $\mathbf{\Gamma}$ consists of m-formulas, $\mathbf{B}$ is a m-formula and $\mathbf{A}$ is 1 or begins with ! . Normal deduction is one without *maximal segments.*

Theorem. Every derivable sequent has a normal deduction.

Proof. Apply Prawitz translation to a cut-free sequent derivation

1 Introduction

We describe a natural deduction system **NDIL** for the second order intuitionistic linear logic which admits normalization and has a subformula property.

For the logic of symmetric monoidal closed categories (called later !-free multiplicative linear logic) this was done in [10] and elaborated in [2]. To include additive connectives and quantifiers it is sufficient to add standard intuitionistic natural deduction rules for these connectives. Inclusion of the modality ! uses a device employed in [14] for natural deduction in S4. This device is combined with a construction $<\Gamma>$ introduced in [9], Sections 7,8 of Chapter I. It takes a form $<\Gamma>_{!A}$ here, and can be used in a deduction below formula !A depending on assumptions Γ. An assumption $<\Gamma>_{!A}$ is interpreted as $!A$ till it is used in the last !-introduction rule, and as Γ after that (cf. Section 4). The index !A is needed to block the !-analog of S4-derivable formula $\Box p \& \Box q \rightarrow \Box(p \& q)$.

Section 6 outlines possible treatment of such topics as normalization theorem, assignment of λ-terms to derivations, and uniqueness of a normal form, using ideas from [11, 7] .

As far as we know no other work used the language with something like $<,>$ to deal with !-introduction rule. There are several natural deduction systems equivalent to ours with respect to derivable formulas (of suitable sublanguage). All of them use sequents $\Gamma \vdash A$, where Γ is a multiset of formulas and A is a formula, plus an assignment of λ-terms in an extended language.

S. Valentini considered in [17] propositional formulas (with additive constants $\top, 0$ for *true, false*). The introduction rules in [17] are essentially the same as ours, except $!I$ which is the same as in the sequent calculus **GIL** below. Elimination rules are modeled after $\otimes E$. For example $\& E$, $!E$ and contraction are of the form:

$$\frac{\Gamma \vdash A \& B \quad A, \Sigma \vdash C}{\Gamma, \Sigma \vdash C} \qquad \frac{\Gamma \vdash !A \quad A, \Sigma \vdash C}{\Gamma, \Sigma \vdash C} \qquad \frac{\Gamma \vdash !A \quad !A, !A, \Sigma \vdash C}{\Gamma, \Sigma \vdash C}$$

Weakening rule is the same as ours.

Normalization steps in [17] are β-conversions for all connectives, but no permutative conversions are postulated. Corresponding normalization theorem provides $\oplus$-property, but not subformula property. It may be difficult to provide a treatment of permutative conversions in this framework.

Natural deduction system considered by P. Lincoln and J. Mitchell in [8] is essentially the same as in [17] plus cut.

A. Troelstra [16] works essentially in the language $\{!, \multimap, \otimes, 1\}$. The rules for all connectives except ! and weakening are essentially the same as ours. The rules for ! are inspired by D. Prawitz' [14] rules for S4, and hence are similar to our rules. The treatment of contraction is rather complicated and prevented extension to the additive fragment (cf. Concluding remarks in [16]). The most essential difference from our treatment is that assumptions are indexed by deductions and can be contracted only if these indices are identical.

S. Ronchi della Rocca and L. Roversi [15] consider the language $\{!, \multimap, \oplus, \otimes, \&\}$. The natural deduction rules for all connectives except $!I$ are essentially the same as ours with more essential differences in the elimination rules for $\oplus, \otimes$. Structural rules are weakening and contraction for the formulas of the form $!A$ combined with other rules in the manner of [6]. The rule $!I$ is the same as $\vdash!$ in the sequent calculus. Normalization steps are β-conversions. There are no commutative conversions and hence only partial subformula property.

The authors of [3] consider multiplicative connectives including ! with !-elimination and contraction similar to $\otimes$-elimination.

Abramsky in [1] does not formulate a natural deduction system explicitly, and his term assignment for the antecedent rules in sequent derivations closely follows the structure of these rules.

This paper is a result of discussions with A. Troelstra and S. Ronchi della Rocca at the meeting 'Mathematical Logic', Mathematical Institute Oberwolfach, April 1995 and P. Lincoln and A. Scedrov at Stanford in May. First version was presented at Logic Lunch, Stanford University.

2 A sequent formulation GIL

This is standard. A, B, D stand for the formulas, Γ, Σ for multisets of formulas.

Axioms:

$$A \vdash A \qquad \vdash 1$$

Inference rules

$$\vdash \& \quad \frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \& B} \qquad \& \vdash \quad \frac{A, \Gamma \vdash D}{A \& B, \Gamma \vdash D} \qquad \& \vdash \quad \frac{B, \Gamma \vdash D}{A \& B, \Gamma \vdash D}$$

$$\vdash \otimes \quad \frac{\Gamma \vdash A \quad \Sigma \vdash B}{\Gamma, \Sigma \vdash A \otimes B} \qquad \otimes \vdash \quad \frac{A, B, \Gamma \vdash D}{A \otimes B, \Gamma \vdash D}$$

$$\multimap \vdash \quad \frac{\Gamma \vdash A \quad B, \Sigma \vdash D}{(A \multimap B), \Gamma, \Sigma \vdash D} \qquad \vdash \multimap \quad \frac{A, \Gamma \vdash B}{\Gamma \vdash (A \multimap B)}$$

$$\vdash \oplus \quad \frac{\Gamma \vdash A}{\Gamma \vdash (A \oplus B)} \qquad \vdash \oplus \quad \frac{\Gamma \vdash B}{\Gamma \vdash (A \oplus B)} \qquad \oplus \vdash \quad \frac{A, \Gamma \vdash D \quad B, \Gamma \vdash D}{A \oplus B, \Gamma \vdash D}$$

$$\vdash! \; \frac{!\Gamma \vdash A}{!\Gamma \vdash !A} \qquad ! \vdash \quad \frac{A, \Gamma \vdash D}{!A, \Gamma \vdash D} \qquad \frac{\Gamma \vdash \mathbf{1} \quad \Sigma \vdash B}{\Gamma, \Sigma \vdash B} \quad \mathit{weak} \quad \frac{\Gamma \vdash D}{!A, \Gamma \vdash D}$$

$$\vdash \exists \quad \frac{\Gamma \vdash A[t]}{\Gamma \vdash \exists x A} \qquad \exists \vdash \quad \frac{A[y], \Gamma \vdash D}{\exists x A[x], \Gamma \vdash D}$$

$$\vdash \forall \quad \frac{\Gamma \vdash A[y]}{\Gamma \vdash \forall x A[x]} \qquad \forall \vdash \quad \frac{A[t], \Gamma \vdash D}{\forall x A[x], \Gamma \vdash D}$$

$$\mathit{Contraction} \quad \frac{!A, !A, \Gamma \vdash D}{!A, \Gamma \vdash D}$$

with suitable terms t and variables y.

All inference rules are by the definition invariant under permutation of formulas in the antecedents.

3 A Natural Deduction System NDIL

m-$formula$ $:= !A \mid 1 \mid\ <$ list of formulas and m-formulas $>_{!A}$

where A is arbitrary formula.

Example. $< p \oplus q, p, < p, q >_{!(r\&s)} >_{!(p\otimes q)}$

A, B, C, D stand for the formulas, $\Gamma, \Delta, \Sigma, Pi$ for multisets of formulas and m-formulas.

Axioms:

$$A \vdash A \qquad\qquad \vdash 1$$

Inference rules

$$\&I\ \frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A\&B} \qquad \&E\ \frac{\Gamma \vdash A\&B}{\Gamma \vdash A} \qquad \&E\ \frac{\Gamma \vdash A\&B}{\Gamma \vdash B}$$

$$\multimap E\ \ \frac{\Gamma \vdash (A \multimap B) \qquad \Sigma \vdash A}{\Gamma, \Sigma \vdash B} \qquad \multimap I\ \ \frac{A, \Gamma \vdash B}{\Gamma \vdash (A \multimap B)}$$

$$\oplus I\ \ \frac{\Gamma \vdash A}{\Gamma \vdash (A \oplus B)}\ \oplus I\ \ \frac{\Gamma \vdash B}{\Gamma \vdash (A \oplus B)}$$

$$\oplus E\ \ \frac{\Sigma \vdash A \oplus B \quad A, \Gamma \vdash D \quad B, \Gamma \vdash D}{\Sigma, \Gamma \vdash D}$$

$$\otimes I\ \frac{\Gamma \vdash A \quad \Sigma \vdash B}{\Gamma, \Sigma \vdash A \otimes B} \qquad \otimes E\ \ \frac{\Gamma \vdash A \otimes B \quad A, B, \Sigma \vdash C}{\Gamma, \Sigma \vdash C}$$

$$<>I\ \ \frac{\Gamma \vdash !A}{< \Gamma >_{!A} \vdash !A} \qquad !E\ \frac{\Gamma \vdash !A}{\Gamma \vdash A} \qquad !I\ \frac{\mathbf{\Gamma} \vdash A}{\mathbf{\Gamma} \vdash !A} \qquad <>E\ \ \frac{< \Gamma >_{!B}, \Sigma \vdash A}{\Gamma, \Sigma \vdash A}$$

$$weak\ \ \frac{\Gamma \vdash \mathbf{A} \quad \Sigma \vdash B}{\Gamma, \Sigma \vdash B} \qquad c\ \ \frac{\mathbf{B}, \mathbf{B}, \Gamma \vdash D}{\mathbf{B}, \Gamma \vdash D}$$

provided $\mathbf{\Gamma}$ consists of m-formulas, $\mathbf{B}$ is a m-formula and $\mathbf{A}$ is 1 or begins with ! .

$$\exists I\ \ \frac{\Gamma \vdash A[t]}{\Gamma \vdash \exists x A} \qquad \exists E\ \ \frac{\Sigma \vdash \exists x A[x] \quad A[y], \Gamma \vdash D}{\Gamma, \Sigma \vdash D}$$

$$\forall I\ \ \frac{\Gamma \vdash A[y]}{\Gamma \vdash \forall x A[x]} \qquad \forall E\ \ \frac{\Gamma \vdash \forall x A[x]}{\Gamma \vdash A[t]}$$

for first- and second-order quantifiers and suitable terms t.

Example 1. $!p\&q \vdash !!p$

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{!p\&q \vdash !p\&q}{!p\&q \vdash !p}}{<!p\&q>_{!p} \vdash !p}}{<!p\&q>_{!p} \vdash !!p}}{!p\&q \vdash !!p}}{}$$

Example 2. $!p\&!q \vdash !(p\&q)$ is not derivable in **NDIL** since it is not derivable in **GIL**. The following figure cannot be turned into a derivation, since one has to assign a subscript $!p$ to $<!p\&!q>$in the left branch, and the subscript $!q$ in the right branch, so that the final $\&I$-inference is blocked.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{!p\&!q \vdash !(p\&!q)}{!p\&!q \vdash !p}}{<!p\&!q> \vdash !p}}{<!p\&!q> \vdash p} \qquad \dfrac{\dfrac{\dfrac{!p\&!q \vdash !(p\&!q)}{!p\&!q \vdash !q}}{<!p\&!q> \vdash !q}}{<!p\&!q> \vdash q}}{<!p\&!q> \vdash p\&q}}{<!p\&!q> \vdash !(p\&q)}}{!p\&!q \vdash !(p\&q)}$$

4 Soundness of Natural Deduction Rules

All rules except $!I$ are obviously sound if one simply drops all $<>$. Hence it is sufficient to replace $!I$ by the following rule

$$(cut!)\dfrac{\Gamma_1 \vdash !A_1,\ \dots,\ \Gamma_n \vdash !A_n \quad !A_1,\dots,!A_n,!\Sigma \vdash D}{\Gamma_1,\dots,\Gamma_n,!\Sigma \vdash !D} \tag{1}$$

which does not involve $<>$ explicitly and is obviously sound: it is a combination of cut and $\vdash!$ rule of the sequent formulation.

Lemma 1 *The rule* $!I$ *is redundant in* **NDIL** $+ (cut!)$

Proof: induction on the number of $!I$-inferences in a given deduction d. The replacement begins with an uppermost occurrence of $!I$ in d. A figure

$$\begin{array}{c}\dfrac{\Gamma_i \vdash !A_i}{<\Gamma_i>_{!A_i} \vdash !A_i}\\ | \\ \dfrac{<\Gamma_1>_{!A_1},\dots,<\Gamma_i>_{!A_i},\dots,<\Gamma_n>_{!A_n},!\Sigma \vdash D}{<\Gamma_1>_{!A_1},\dots,<\Gamma_i>_{!A_i},\dots,<\Gamma_n>_{!A_n},!\Sigma \vdash !D}\end{array}$$

is replaced in the following way: all predecessors of occurrences of $<\Gamma_i>_{!A_i}$ are replaced by $!A_i$, and the explicitly shown $!I$ by *cut!*:

$$\dfrac{<\Gamma_1>_{!A_1} \vdash !A_1,\dots,<\Gamma_n>_{!A_n} \multimap !A_n \quad \begin{array}{c} !A_i \vdash !A_i \\ | \\ !A_1,\dots,!A_i,\dots,!A_n,!\Sigma \vdash D\end{array}}{<\Gamma_1>_{!A_1},\dots,<\Gamma_n>_{!A_n},!\Sigma \vdash !D}$$

Theorem 1 *If a* $<,>$*-free sequent is derivable in* **NDIL**, *then it is derivable in* **GIL**.

Proof. If a sequent is derivable in **NDIL**$+(cut!)$, eliminate $!I$ by the Lemma 1 and erase $<,>$. All remaining rules are derivable in **GIL** $+$ *cut!*. Use cut-elimination theorem for **GIL** (cf. [5]).

5 Construction of a Normal Deduction by Prawitz translation

Recall that a *segment* in a natural deduction is series of consecutive sequents with *unchanged* succedent:

$$\begin{array}{c}\Gamma \vdash M \\ | \\ | \\ \Sigma \vdash M\end{array}$$

In other words, a segment consists of one sequent or of applications of the rules $\oplus E, \otimes E, <> I, \exists E$ and structural rules. A segment is *maximal* if its uppermost sequent is a conclusion of an introduction rule, and its last sequent ia the major premise of an elimination rule. A natural deduction is *normal* if it has no maximal segments.

For every sequent derivation $d : \Gamma \vdash D$ we define a normal natural deduction $P(d) : \Gamma \vdash D$ by induction on d.

Recall some properties of natural deductions.

P1. Every antecedent formula A of the endsequent is traceable up the deduction to an antecedent formula of an axiom. This is pictured as follows:

$$\begin{array}{c} A \vdash A \\ | \\ A, \Theta \vdash E \\ | \\ | \\ A, \Gamma \vdash D \end{array}$$

P2. Inference rules are preserved by substitution of an arbitrary list Γ of formulas for all predecessors of an antecedent formula (occurrence) A in the endsequent with the standard provisos for eigenvariables and additional proviso for !I: if A is 1 or begins with !, then Γ consists of m-formulas.

P3. If a natural deduction is normal and ends in an elimination rule for a connective, then the main formula (containing the eliminated connective) is a strictly positive subformula of the antecedent of the major premise.

Inductive Definition of the transformation P.

If the sequent derivation d is the axiom,then $P(d) := d$. This deduction is obviously normal.

If d ends in the succedent rule, then apply induction hypothesis to premises and add the same rule. Contraction and weakening are treated similarly, but !-weakening is replaced by

$$\frac{!A \vdash !A \qquad \Gamma \vdash D}{!A, \Gamma \vdash D}$$

Since the last rule is not an elimination, no maximal segments are introduced, hence normality is preserved.

Assume now that d ends in an antecedent rule L for a connective $\mathbf{c}$. If $\mathbf{c}$ is $\oplus$ or $\exists$, then the axiom over the main formula of L is added as an additional premise to deductions obtained by the induction hypothesis, resulting in a $\mathbf{c}$-elimination inference. For example, $\otimes \vdash$ is transformed into

$$\frac{A \otimes B \vdash A \otimes B \quad A, B, \Gamma \vdash D}{A \otimes B, \Gamma \vdash D}$$

Since the major premise of the final elimination rule is an axiom, this rule does not create a maximal segment, hence normality is preserved. The same argument works in the next two cases.

In the remaining cases write the endsequent as

$$M, \Gamma \vdash D \tag{2}$$

where M is the main formula of L. Let F be an antecedent side formula of the rule L, and let

$$F, \Delta \vdash D \tag{3}$$

be the corresponding premise of the rule L.

Let M' be M if F is not 1 and does not begin with !, and let M' be $< M >$ otherwise. Note that in the former case the sequent

$$M' \vdash F \quad i.e. \quad < M >_F \vdash F \tag{4}$$

is derivable by an elimination rule and possibly $<>$-rule from an axiom $M \vdash M$ if $L \neq \vdash \multimap$. Replace all predecessors of F by M' in the deduction of (3) obtained by the induction hypothesis. By P2 this preserves all the rules, hence only axioms

$$F \vdash F$$

are to be checked. Use (4). For example, if

$$M =!A\&B \quad and \quad F =!A$$

then the deduction of $!A, \Gamma \vdash D$ existing by the induction hypothesis is of the form

$$\begin{array}{c}
!A \vdash !A \\
| \\
\dfrac{!A, \boldsymbol{\Theta} \vdash E}{!A, \boldsymbol{\Theta} \vdash !E} \\
| \\
| \\
!A, \Gamma \vdash D
\end{array}$$

Prawitz transformation produces the following deduction:

$$\begin{array}{c}
\dfrac{\dfrac{!A\&B \vdash !A\&B}{!A\&B \vdash !A}}{<!A\&B>_{!A} \vdash !A} \\
| \\
\dfrac{<!A\&B>_{!A}, \boldsymbol{\Theta} \vdash E}{<!A\&B>_{!A}, \boldsymbol{\Theta} \vdash !E} \\
| \\
| \\
<!A\&B>_{!A}, \Gamma \vdash D
\end{array}$$

Finally, let $L =\vdash \multimap$, i.e.

$$\frac{\Sigma \vdash A \quad F, \Delta \vdash D}{(A \vdash F), \Sigma, \Delta \vdash D}$$

Then we replace all predecessors of F in a deduction of (3) obtained by the induction hypothesis by the list Σ', i.e. by $\Sigma, A \multimap F$ or $< \Sigma, A \multimap F >_F$ depending of the form of F. All rules are preserved, and the axioms $F \vdash F$ are replaced first by the sequent

$$\Sigma' \vdash F,$$

and then by the following derivation (written for the case when F is 1 or begins with !)

$$\frac{\dfrac{\Sigma \vdash A \qquad A \multimap F \vdash A \multimap F}{\Sigma, A \multimap F \vdash F}}{< \Sigma, A \multimap F >_F \vdash F}$$

This concludes the description of P and proves the following

Theorem 2 *Every sequent derivable in* **GIL** *has a normal deduction in* **NDIL**.

6 Further work

6.1 Normalization Theorem

Our proof of the Theorem 2 provides a method for constructing a normal form of a natural deduction. There are two further methods of obtaining a result more similar to what one has for λ-calculus. It is easy to define reductions (normalization steps) for natural deductions. There seems to be no obstacles to extension of Girard's method of computability predicates to establish a strong normalization theorem: every sequence of reductions terminates in a normal form.

Another proof of a normalization theorem would contain 3 steps.

1. Given a (non-normal) natural deduction d, apply Prawitz [14] translation (call it $\mathcal{G}$) to transform d into a derivation $\mathcal{G}(d)$ in the sequent calculus **GIL**. The latter contains cuts exactly at the places where d contains maximal segments.

2. Use normalization theorem for **GIL** to obtain reduction sequence

$$s_0 = \mathcal{G}(d)\ red\ s_1 \ldots red\ s_n \quad \mathrm{cut-free} \tag{5}$$

3. Verify that each reduction in 5 is mapped by the Prawitz translation P into a series of reduction for **NDIL**-deduction: $P(s_i)\ red\ P(s_{i+1})$ for every $i < n$. Here one should follow [13], and not earlier work by Zucker [18]. Finally, $P(\mathcal{G}(d)) = d$.

6.2 Term assignment

We still prefer original treatment of multiplicative rules in [10] where the set of terms for the $\{\multimap, \otimes\}$-fragment was a subset of standard λ-terms with paring. Subsequent treatments in the framework of the linear logic introduced a new construction closely modeling $\otimes$-antecedent rule. Extension of the former assigment to the rules for remaining connectives does not present any difficulties: additive connectives and quantifiers are treated essentially as the corresponding intuitionistic connectives, and $!I$ transforms a term t into $!t$.

6.3 Uniqueness of a Normal Form

6.3.1 !-free Fragment

$\otimes$-elimination rule introduces into natural deduction the same kind of non-uniqueness as the $\otimes$-antecedent rule of the sequent calculus. This can be dealt with by a device going back to “central maps” of [7]. It introduces a kind of eqivalence relation which corresponds to the commutativity of $\otimes$ and dispences with the permutations of adjacent $\otimes$-inferences.

Let’s make some definite arrangements about structural rules. For example, move weakenings up to the axioms choosing the rightmost premise when there is a choice, i.e. for $\multimap E, \otimes E, \exists E$. This means that corresponding conversions are postulated and a weakening is *normal* if its right premis is an axiom.

Similarly, contractions can be moved maximally downward as in [6] or maximally upward as $\otimes E$ below.

Next we move all $\otimes E$-inferences North-East till their side formulas are separated.

Definition. A two-premise inference L *separates* (side formulas A_1, A_2 of) the $\otimes E$-inference if they are situated as follows:

$$\dfrac{\Gamma \vdash A_1 \otimes A_2 \qquad \begin{array}{c} \dfrac{A_i, \Delta' \vdash D' \qquad A_j, \Delta'' \vdash D''}{A_1, A_2, \Delta \vdash D}L \\ |\quad| \\ A_1, A_2, \Sigma \vdash C \end{array}}{\Gamma, \Sigma \vdash C}\otimes E$$

Here L is $\multimap E, \otimes E$ or $\exists E$, and $\{i, j\} = \{1, 2\}$. The definition of separation for a contraction inference is similar.

Definition. An inference L *immediately separates* inference M if L separates M as well as all inferences between L and M.

$$\begin{array}{c} |/ \\ L \\ / \\ |/ \\ L' \\ |/ \\ L \end{array}$$

Endpiece of a derivation is a sequence of inferences beginning with a rule L different from $\otimes E$ and proceeding through the minor (right-hand side) premises of $\otimes E$-inferences down to the endsequent. Endpiece is *normal* if its uppermost inference L immediately separates all remaining inferences.

A derivation is *normal* if it is cut-free, has structural inferences in place and the endpiece of every subderivation is normal.

Conjecture. If natural deductions d, d' are normal and interconvertible under a suitable set of conversions, then they differ only by an order of $\otimes E$ inferences in the endpieces.

6.3.2 !-rules

Let us restate the rules in [16] in our language. m-formulas to be added to the standard language would be of the form $< \Gamma >_t$ where t is a (term assigned) deduction of $\Gamma \vdash !A$. Main new instance of a contraction rule in [16] is of the form

$$\frac{< \Gamma >_t, < \Gamma >_t, \Sigma \vdash D}{< \Gamma >_t, \Sigma \vdash D} c1$$

where the subscript is one and the same in both contracted formulas of the premise. Note that the latter restriction is violated in our formulation: our contraction rule can be stated as

$$\frac{< \Gamma >_{t_1}, < \Gamma >_{t_2}, \Sigma \vdash D}{< \Gamma >_{t_i}, \Sigma \vdash D} c2$$

with $t_1, t_2 : \Gamma \vdash !A$ and $i \in \{1, 2\}$. Both rules generate one and the same class of derivable sequents, but to reduce $c2$ to $c1$ one has to identify differnt deductions t_1, t_2 , and this hardly leaves hope for a unique normal form. This probably will force returning back to the rules without $<, >$ listed in the introduction.

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